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The cosmological dependence of weak interactions

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Abstract. A model for the cosmological time dependence of weak interactions is discussed and some experimental tests suggested.

1. Introduction

In this paper a model is described which suggests a link between gravitational and weak interactions. Thus any time dependence in the gravitational interaction appears also in the weak interactions. In § 2 the model is presented and the specific form that the time variation of the weak interactions takes is developed. In § 3 some of the consequences of the model involving laboratory neutrinos are discussed. Section 4 describes a possible program for testing the model by the detection of cosmic neutrinos. We conclude with § 5 in which some further speculations about the time development of physical laws are made.

2. The cosmological model

The idea that interactions may change with time stems from a paper by Dirac (1937) in which the gravitational constant was treated as time dependent. However, it was later realized that, as originally expressed, his hypothesis contradicted the principle of covariance. Jordan (1961, *Problems in Gravitation* unpublished) and others (Dicke 1963) have since produced a way of circumventing this difficulty by introducing a scalar field $\phi(x)$ into the theory. Recently this ϕ field has been used to develop singularity-free cosmological models (Novello 1971, unpublished) by employing a nonlinear Lagrangian in a scalar-tensor theory of gravitation. To obtain this well-behaved form of the Universe the ϕ field must have a regular minimum at $t = 0$

$$\phi(t) \sim \phi_0 + \phi_1 t^2 + \dots \quad (\text{for small } t) \quad (1)$$

and go to a constant Q for large values of t

$$\phi(t) \xrightarrow[t \rightarrow \infty]{} Q. \quad (2)$$

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Now, the riemannian structure of space–time implies that the generalized γ which define the metric tensor by the anticommutation relation

$$\{\gamma_\alpha(x), \gamma_\beta(x)\} = 2g_{\alpha\beta}(x)\mathbb{1} \tag{3}$$

obey the equation

$$\gamma_{\alpha||\beta}(x) = \sigma[U_\beta(x), \gamma_\alpha(x)] \tag{4}$$

where σ is a constant, $U_\beta(x) \equiv \gamma_\beta(x)(1 + \gamma_5(x))$ and double bar ($||$) means covariant derivative, that is

$$\gamma_{\alpha||\beta}(x) = \gamma_{\alpha|\beta}(x) - \Gamma_{\alpha\beta}^\epsilon(x)\gamma_\epsilon(x) + [\tau_\beta(x), \gamma_\alpha(x)]$$

where the single bar means the usual derivative, $\Gamma_{\alpha\beta}^\epsilon$ are the connections of the Riemann space and τ_α are *internal* connections that arise from the permissible generalized gauge transformation

$$\gamma_\alpha(x) \rightarrow \gamma'_\alpha(x) = M(x)\gamma_\alpha(x)M^{-1}(x)$$

for an arbitrary matrix $M(x)$.

Equation (4) is the most general expression consistent with the riemannian structure of the space that can be constructed with the elements of the Clifford algebra without including any arbitrary extra field.

It has previously been shown (Novello 1971) that starting from this evolution operator U_λ (equation (4)) for the generalized Clifford algebra (space–time dependent) one can arrive at a modified class of Einstein’s equations that relate the riemannian contracted curvature tensor with $\mathbb{R}_{\alpha\beta}$ the curvature of the internal space

$$R_{\alpha\beta}(x)\gamma^\alpha(x) + [\mathbb{R}_{\alpha\beta}(x), \gamma^\alpha(x)] = 0. \tag{5}$$

This immediately suggests a link between gravitation and weak interactions because, given the form of $U_\lambda(x)$, the only nontrivial interaction Lagrangian which can be constructed from U_λ and spinor fields is the current–current interaction

$$\mathcal{L}_I = \frac{G}{\sqrt{2}} J_\beta J^\beta \tag{6}$$

where

$$J_\beta(x) = \bar{\psi}(x)\gamma_\beta(x)(1 + \gamma_5(x))\psi(x). \tag{7}$$

The above consideration induces us to propose that the modified form of Dirac’s idea should be applied not only with respect to the gravitational interaction but also to the weak interactions. In a homogeneous and isotropic cosmological model—such as the one we are considering—we shall see that the influence of cosmology on the weak interactions produces a time-dependent weighting of the axial vector current relative to to the vector current. A direct way to do this is to consider the Lagrangian (6) and compare it with the usual flat-space Lagrangian

$$\mathcal{L}' = \frac{G}{\sqrt{2}} j_\alpha j^\alpha \tag{8}$$

where

$$j_\alpha = \bar{\psi}\gamma_\alpha(1 + \gamma_5)\psi \tag{9}$$

(γ_α and γ_5 being *constant* Dirac matrices). In the particular type of universe we are considering we may write

$$\gamma_\alpha(x) = F(\alpha, x)\gamma_\alpha. \tag{10}$$

Indeed, from (3) and because in the co-moving system of coordinates

$$ds^2 = dt^2 - F_1(dx^1)^2 - F_2(dx^2)^2 - F_3(dx^3)^2 \tag{11}$$

it follows that :

$$\begin{aligned} \gamma_0(x) &= \gamma_0 \\ \gamma_1(x) &= F_1^{1/2}\gamma_1 \\ \gamma_2(x) &= F_2^{1/2}\gamma_2 \\ \gamma_3(x) &= F_3^{1/2}\gamma_3. \end{aligned} \tag{12}$$

Substituting (12) and (11) into (7) yields for the interaction defined by (6) the expression

$$\begin{aligned} \mathcal{L}_1 = \frac{G}{\sqrt{2}} \left\{ \bar{\psi}\gamma_0(1 + \epsilon(x)\gamma_5)\psi \right\} \left\{ \bar{\psi}\gamma^0(1 + \epsilon(x)\gamma_5)\psi \right\} \\ + \left\{ \bar{\psi}\sqrt{F_1}\gamma_1(1 + \epsilon(x)\gamma_5)\psi \right\} \left\{ \bar{\psi}\frac{1}{\sqrt{F_1}}\gamma^1(1 + \epsilon(x)\gamma_5)\psi \right\} + \dots \end{aligned}$$

which shows that the only effective modification of generalizing to the $\gamma_\alpha(x)$ functions is a space–time weighting of the axial vector current relative to the vector current (the vector current modification being absorbed in \mathcal{L}_1 by the modified space–time metric tensor). Thus we may write the weak leptonic current as

$$J_\alpha = \bar{\psi}\gamma_\alpha(1 + \epsilon(x)\gamma_5)\psi \tag{13}$$

where $\epsilon(x)$ is a function of $\phi(x)$ and the simplest assumption would be that they are linearly related, that is

$$\epsilon(x) = \frac{1}{Q}\phi(x) \tag{14}$$

whence the maximal violation of parity in the V–A theory is reached only at asymptotic cosmological time.

3. Consequences for the weak interactions

The model has two obvious consequences: (i) Since we do not exist at asymptotic cosmological time, the present leptonic weak current does not violate parity maximally. (ii) Produced neutrinos and antineutrinos are admixtures of both *left* and *right* polarized states. The ratio of the admixture depends upon their (cosmological) time of creation.

The first consequence can best be tested by a very accurate laboratory measurement of the Michel parameter ρ in μ meson decay. Let us define a parameter δ by writing the present weak leptonic current as

$$J_\alpha = \bar{\psi}\gamma_\alpha\{1 + (1 - \delta)\gamma_5\}\psi \tag{15}$$

that is, $\epsilon(t_0) = 1 - \delta$ where t_0 is our present cosmological time. The $V-A$ theory† appears in the limit $\delta = 0$. If we then neglect, for the moment, all masses involved except for the μ meson mass and neglect the calculable radiative corrections, we find that

$$\rho = \frac{3(1 - 2\delta + \frac{3}{2}\delta^2)}{4(1 - 2\delta + 2\delta^2)} \simeq \frac{3}{4}(1 - \delta^2/2). \tag{16}$$

Since, as we shall see, this may be a very small modification to the usual value of $\frac{3}{4}$, we have recalculated the decay rate dW for a polarized muon retaining the electron mass and the μ neutrino mass (< 1.15 MeV) but neglecting the electron antineutrino mass because of its very low experimental upper limit (< 60 eV). We find to order δ^2

$$\begin{aligned} \frac{dW}{dE d \cos \theta} = & \frac{(1 - \gamma)^2 G^2 (1 - 2\delta + \frac{3}{2}\delta^2) |k|}{24\pi^3} \left\{ E(\mu^2 + e^2 - 2\mu E)(1 + 3\delta^2 - \gamma) \right. \\ & + 2(\mu - E)(\mu E - e^2)(1 + 2\gamma) - 3\nu(\mu E - e^2)\delta(1 + \delta/2) + \frac{\alpha}{2\pi} f(E) \\ & \left. + |k| \cos \theta \left(\mu^2 + 3e^2 - 2\mu E - \gamma(\mu^2 - 3e^2 + 2\mu E) + \frac{\alpha}{2\pi} g(E) \right) \right\} \tag{17} \end{aligned}$$

where E , $|k|$ and e are the electron energy, magnitude of momentum ($|k| = (E^2 - e^2)^{\frac{1}{2}}$) and mass, respectively, while μ and ν are the masses of the muon and muon neutrino, respectively. $\gamma = \nu^2/(\mu^2 + e^2 - 2\mu E)$ and, since the kinematically allowed values for the electron energy run from $E_{\min} = e$ to $E_{\max} = (\mu^2 + e^2 - \nu^2)/2\mu$, it follows that the term $(1 - \gamma)^2$ vanishes as $E \rightarrow E_{\max}$. Thus, as is well known, an accurate determination of the electron energy spectrum will yield at least an upper limit for the muon neutrino mass. The functions $f(E)$ and $g(E)$ represent the effects of radiative corrections. As a first approximation, and in order to obtain an upper limit on δ , we may use the first-order corrections calculated by Kinoshita and Sirlin (1959). Experimentally no disagreement with the $V-A$ theory has yet been found. Indeed, allowing for the above radiative corrections (Bardon *et al* 1965, Derenzo 1969)

$$\rho_{\text{exp}} = 0.75 \left(\frac{1}{2} \right) \pm 0.003. \tag{18}$$

If we allow ourselves up to 1 STD we can set an upper limit to δ of

$$\delta < 0.05. \tag{19}$$

The electromagnetic corrections to μ decay are particularly important, introducing an effective diminution of several per cent in ρ . Thus to determine δ we shall require not only more accurate experiments but also theoretical calculations of second-order radiative corrections‡ (Marshak *et al* 1969).

A similar determination of an upper limit to δ follows from the modification in our model from the expression for the polarization (P_e) of the produced electron in β decay. Following the usual assumption that the nucleons in this process are effectively at rest,

† We of course employ the lepton number-conserving charged currents. Often, however, (and particularly in μ decay) V , A , S , T and P are defined for the 'charge retention' currents. Our modification of the $V-A$ theory corresponds to the appearance of $S-P$ terms in addition to $V-A$ in the charge retention current.

‡ It should also be noted that these second-order corrections will require a cut-off to cope with the ultraviolet divergences as indeed do the first-order radiative corrections if note is taken of the deviation from an exact $V-A$ model.

and integrating over the outgoing antineutrino three-momentum, we find that

$$dW \propto (1 + 3\eta^2)m^2 E_e E_{\bar{\nu}} [(1 - \zeta)\{(1 - \delta/2)^2(1 + v_e) + \delta^2(1 - v_e)/4\} + (1 + \zeta)\{(1 - \delta/2)^2(1 - v_e) + \delta^2(1 + v_e)/4\}] \tag{20}$$

where $\eta = g_A/g_V$ for the hadronic current and where ζ is the electron polarization vector, v_e its relativistic velocity and E_e and $E_{\bar{\nu}}$ the energies of electron and antineutrino, respectively. Thus we find that

$$P_e = \frac{R - L}{R + L} = -v_e(1 - \delta^2/2). \tag{21}$$

Experimentally (Willis and Thompson 1968)

$$P_{e^-}/v_{e^-} = -1.001 \pm 0.008 \tag{22}$$

which, allowing for 1 STD, sets an upper limit to δ of

$$\delta < 0.12. \tag{23}$$

Thus we conclude that although no direct evidence for a nonzero δ exists, the above data only set an upper limit of about one-twentieth on its value.

4. Cosmic neutrinos

As a test of the second consequence listed in § 3 we first note that a neutrino (antineutrino) produced at cosmological time t by a weak current of the form $\bar{\psi}\gamma_\alpha(1 + \epsilon(t)\gamma_5)\psi$ can be written as

$$\psi_{\nu(\bar{\nu})}^{(t)} = \cos \theta(t)\psi_{\nu(\bar{\nu})}^{L(R)} + \sin \theta(t)\psi_{\nu(\bar{\nu})}^{R(L)} \tag{24}$$

where by L and R we mean left and right polarizations, respectively, and where $\tan \theta(t) = (1 - \epsilon(t))/(1 + \epsilon(t))$. If, as is usually assumed, we lived in a world in which $\delta = 0$ and the neutrinos were massless, then the right-(left-)handed polarized neutrinos (antineutrinos) would be completely invisible to any detection apparatus (save possibly one employing the gravitation interaction). Thus the only observable consequences of the model, for the detection of cosmic neutrinos (antineutrinos) on the Earth, would be the effective diminution of the universal Fermi constant G by $\cos \theta(t)$. This could be measured in the *Gedenken* experiment in which all L neutrinos (R antineutrinos) from a particular source are absorbed and counted during a specified time. This provides us with an experimental measurement of the otherwise elusive flux, and together with a measurement of their rate of interaction the ‘effective’ Fermi constant could be deduced. Of course this is impossible in practice because of the very low interaction rate of the neutrinos (antineutrinos), and very few of those entering a laboratory will be detected. It is this very property, however, which in part makes cosmic neutrinos so interesting, for if detected they may well carry information from very distant sources.

A feasible experiment depends upon a nonzero δ , since this will allow us, in principle, to measure both $\psi_{\nu(\bar{\nu})}^{L(R)}$ and $\psi_{\nu(\bar{\nu})}^{R(L)}$. Indeed the rates of interaction of these components, from a particular source, are proportional to $\cos^2 \theta(1 - \delta/2)^2 \times \text{flux}$ and $\sin^2 \theta(\delta^2/4) \times \text{flux}$, respectively (ignoring for simplicity any possible mass for the neutrino). Since the flux is the same in both cases, the ratio of these rates determines $\delta^2 \tan^2 \theta$. This ratio appears,

for example, in the polarization of electrons produced via inverse β decay (for low-momentum transfer):

$$\frac{P_e}{v_e} \simeq \frac{\delta^2 \tan^2 \theta - 4}{\delta^2 \tan^2 \theta + 4}. \quad (25)$$

It is therefore conceivable that some cosmic neutrinos produce unpolarized electrons. To proceed further and try to determine the time dependence of $\theta(t)$ we would require the development of a 'neutrino telescope', that is, an equipment in which the momentum of the incoming neutrino (antineutrino) could be deduced from the outgoing particles of its interaction. It is probable that neutrinos (antineutrinos) from a specific direction are dominated by one source and that the cosmological age of the source will vary with direction. Thus a measurement of the ratio in equation (25) would also vary with direction indicating a time dependence of $\theta(t)$. If in addition these sources could be identified with known radio or optical sources the actual dependence on time of $\theta(t)$ could be found. Such a 'telescope' could also be used to determine accurately any mass of the neutrino by measuring the time delay between neutrinos and photons produced in for example, an exploding star, as has already been suggested (Pontecorvo 1968).

Unfortunately this program, in spite of some optimistic plans (Ginsburg 1971) is hampered by the low energies envisaged (due to the Doppler effect) for cosmic neutrinos, their predicted low flux and the failure up to the present time of attempts to measure the much more plentiful neutrinos expected from the sun.

5. Conclusions

It follows from what has been said in the previous section that the test of the model by direct measurement of cosmic neutrinos is not likely in the near future. There is, however, another consequence of the model which may prove relevant, and that is that because we now predict the existence of a full complement of four neutrinos (and four anti-neutrinos) the usual estimates of the maximum energy density of the neutrino sea in the Universe become doubled.

It is tempting both from the upper limits set on δ and by the very 'unaesthetic' nature of its consequence (the breakdown in the maximal violation of parity) to suggest a link between this effect and CP violation. In this vein CP violation would, like δ itself, be a vanishing phenomenon, and in addition one would expect the two phenomena to be of the same order of magnitude, that is, about 10^{-3} smaller than the weak interaction in general. If this is the case then present experimental accuracies, particularly in μ decay, are not far short of the mark.

We expect no modification in the strength of the electromagnetic charge coupling because of the CVC (conserved vector current) hypothesis relating it to the unmodified vector part of the weak leptonic current. But we might have a parity-violating term of the kind $J_{5\mu} A^\mu$ which, in analogy with the CP-violating term in weak interactions, is a vanishing function with cosmological time.

Finally, since we have discussed the possible time variation in gravitational, weak and even electromagnetic interactions, all linked to the time dependence of the scalar ϕ field, it would be unjust not to contemplate the possible time dependence of the strong interactions themselves (Davies 1972). In general we are advocating a theory in which the physical laws are a function of space-time (appearing constant only locally).

It may in fact well be that the existence of 'unexplained' energy sources in the Universe is just a consequence of our microscopic view of the laws of physics.

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